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# WAVE REGIMES ON A FERROMAGNETIC VISCOUS-FLUID FILM FLOWING DOWN A VERTICAL CYLINDER

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*Axially symmetrical waves on the surface of a ferromagnetic viscous-fluid film flowing down a cylindrical conductor with alternating current are considered. In this case, in addition to the gravitational force, the film is affected by a spatially nonuniform time-dependent magnetic field. The film thickness was assumed to be small compared to the radius of the conductor. In the long-wave approximation, a model equation for the deviation of the film thickness from its undisturbed value is obtained. Some numerical solutions of this equations are reported.*

**1. Formulation of the Problem.** It is known that the substantial difference between flows of ferromagnetic and normal fluids is due to the volume magnetic force acting in the direction normal to the stream velocity [1, 2]. This force largely affects stability of the flow. In its various statements, the problem about a film flow of a ferromagnetic fluid was studied in many works. For instance, Demekhin et al. [3] considered the problem about the wave regime of a thin layer of a magnetic fluid flowing down a vertical conductor with direct current at moderate Reynolds numbers. In the long-wave approximation, the authors [3] obtained a system of nonlinear differential equations describing the propagation of finite-amplitude axisymmetric disturbances. Some numerical solutions of the system were constructed.

In the present work, we consider the effect of the time-dependent magnetic force on the wave regimes of the flow.

A thin-film flow of a viscous ferromagnetic fluid flowing down a vertical cylindrical conductor in the gravity field is examined. The conductor carries an alternating current $I_0 \cos(\omega_0 t + \phi)$. The law of magnetization of the magnetic fluid is assumed to be linear. We consider an axially symmetrical case, in which no disturbances of the magnetic field $\boldsymbol{H}$ and no magnetic jump of pressure occur on the free surface of the fluid. In the long-wave approximation and in the case of low rates of the fluid flow (Reynolds numbers of the order of unity), this problem can be reduced to solving one nonlinear differential equation for the deviation of the film thickness from its undisturbed value.

In the quasi-stationary approximation, the system of the Navier–Stokes equations that describes the film flow has the form [1, 2]

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \nu\Delta\boldsymbol{v} + \boldsymbol{g} + \frac{\boldsymbol{f}}{\rho}, \qquad \operatorname{div}\boldsymbol{v} = 0. \tag{1}$$

Here $\boldsymbol{v}$ is the velocity, $p$ is the pressure, $\rho$ is the density, $\nu$ is the kinematic viscosity, $t$ is the time, $\boldsymbol{g}$ is the free-fall acceleration, $\boldsymbol{f} = \mu_0(\boldsymbol{M}\nabla)\boldsymbol{H}$ is the force acting on a unit volume of the ferromagnetic fluid from the side of the magnetic field, $\mu_0$ is the magnetic constant, $\boldsymbol{H}$ is the magnetic-field intensity, and $\boldsymbol{M}$ is the magnetization of the fluid.

We introduce a cylindrical coordinate system in which the $z$ axis directed along the gravitational force is the centerline of the conductor of radius $R$. The radial coordinate of the boundary of the undisturbed free surface of the ferromagnetic fluid is $r = R + h_0$ ($h_0$ is the thickness of the undisturbed film).

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The magnetic field produced by the linear current outside the conductor is

$$r > R: \qquad \boldsymbol{H} = (0, H_\varphi, 0), \quad H_\varphi = I/(2\pi r) = I_0 \cos(\omega_0 t + \phi)/(2\pi r).$$

Since in the flow region $\operatorname{rot}\boldsymbol{H} = 0$ and a linear law of magnetization $\boldsymbol{M} = \chi\boldsymbol{H}$ is assumed ($\chi$ is the magnetic susceptibility of the ferromagnetic fluid), then

$$\boldsymbol{f} = \mu_0(\boldsymbol{M}\nabla)\boldsymbol{H} = \mu_0\chi(\boldsymbol{H}\nabla)\boldsymbol{H} = \mu_0\chi(\nabla\boldsymbol{H}^2/2 - [\boldsymbol{H}, \operatorname{rot}\boldsymbol{H}]) = \mu_0\chi\nabla\boldsymbol{H}^2/2.$$

At the surface of the cylinder, we adopt the boundary condition

$$r = R: \qquad \boldsymbol{v} = 0, \tag{2}$$

and at the free surface of the film we use the kinematic condition

$$\frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial z} = v \tag{3}$$

and dynamic conditions

$$r = R + h_0 + h: \qquad (p - \sigma/R_*)n_i - \tau_{ik}^{(1)} n_k = p_0 n_i - \tau_{ik}^{(2)} n_k. \tag{4}$$

Here $u$ and $v$ are, respectively, the $z$- and $r$-components of the flow velocity, $p$ is the pressure in the fluid; $p_0$ is the ambient pressure, $n_i$ are the components of the normal $\boldsymbol{n} = (n_z,\, n_r) = (-\partial h/\partial z, 1)/\sqrt{1 + (\partial h/\partial z)^2}$,

$$\frac{1}{R_*} = \frac{[1 + (\partial h/\partial z)^2]/(R + h_0 + h) - \partial^2 h/\partial z^2}{[1 + (\partial h/\partial z)^2]^{3/2}}$$

is the mean curvature, $\tau_{ik} = \sigma_{ik} + H_i B_k - \mu_0 H^2 \delta_{ik}/2$ is the stress tensor, which includes the viscous-stress tensor $\sigma_{ik}$ and the Maxwell stress tensor [the superscripts (1) and (2) in formula (4) refer to the fluid and to the ambient medium], and $\boldsymbol{B} = \mu_0(1 + \chi)\boldsymbol{H}$ is the magnetic induction.

System (1) with the boundary conditions (2)–(4) admits a solution with a cylindrical free surface of uniform thickness

$$u(r) \equiv V = \frac{g}{2\nu}\Big((R + h_0)^2 \ln\frac{r}{R} - \frac{r^2 - R^2}{2}\Big), \quad P(r) = p_0 + \frac{\sigma}{R + h_0} + \frac{\mu_0\chi I^2}{8\pi^2}\Big(\frac{1}{r^2} - \frac{1}{(R + h_0)^2}\Big).$$

It can easily be shown, however, that such a flow is unstable against infinitesimal perturbations and, as these perturbations further develop, this flow becomes wavy.

The purpose of the present study is to derive a simple model for nonlinear wave regimes in a falling film of a ferromagnetic fluid.

Let $V_0$ be the velocity of the free surface of the undisturbed film of thickness $h_0$, and $\lambda$ be the characteristic streamwise length of the perturbation. For convenience, we may introduce, instead of the radial coordinate $r$, the quantity $r - R$, which hereafter will be designates as $r$.

Using the parameters $\lambda/V_0$, $V_0$, and $\rho g h_0$ as time, velocity, and pressure scales, and $h_0$ and $\lambda$ as spatial scales in the $r$- and $z$-directions, respectively, we write the equations of the film flow in the nondimensional form:

$$\varepsilon\,\frac{\partial u}{\partial t} + \varepsilon u\,\frac{\partial u}{\partial z} + v\,\frac{\partial u}{\partial r} = -\frac{\varepsilon}{\mathrm{Fr}}\,\frac{\partial p}{\partial z} + \frac{\delta}{\mathrm{Re}(1 + \delta r)}\,\frac{\partial u}{\partial r} + \frac{1}{\mathrm{Re}}\,\frac{\partial^2 u}{\partial r^2} + \frac{\varepsilon^2}{\mathrm{Re}}\,\frac{\partial^2 u}{\partial z^2} + \frac{1}{\mathrm{Fr}},$$

$$\varepsilon\,\frac{\partial v}{\partial t} + \varepsilon u\,\frac{\partial v}{\partial z} + v\,\frac{\partial v}{\partial r} = -\frac{1}{\mathrm{Fr}}\,\frac{\partial p}{\partial r} + \frac{\delta}{\mathrm{Re}(1 + \delta r)}\,\frac{\partial v}{\partial r}$$

$$+ \frac{1}{\mathrm{Re}}\,\frac{\partial^2 v}{\partial r^2} + \frac{\varepsilon^2}{\mathrm{Re}}\,\frac{\partial^2 v}{\partial z^2} - \frac{\delta^2}{\mathrm{Re}(1 + \delta r)^2}\,v - \frac{\mathrm{B}\,\mathrm{We}\,\delta^2}{\mathrm{Fr}(1 + \delta r)^3}\,\cos^2(\omega t + \phi), \tag{5}$$

$$\varepsilon\,\frac{\partial u}{\partial z} + \frac{\delta}{1 + \delta r}\,v + \frac{\partial v}{\partial r} = 0.$$

Here $\mathrm{Re} = V_0 h_0/\nu$ is the Reynolds number, $\mathrm{Fr} = V_0^2/(g h_0)$ is the Froude number, $\mathrm{We} = \sigma/(\rho g h_0^2)$ is the Weber number, $\sigma$ is the surface-tension coefficient, $\mathrm{B} = \mu_0\chi I_0^2/(4\pi^2\sigma R)$ is the Bond magnetic number, $\omega = \omega_0\lambda/V_0$, $\varepsilon = h_0/\lambda$, and $\delta = h_0/R$.

The boundary conditions are

$$r = 0\colon \quad \boldsymbol{v} = 0; \tag{6}$$

$$r = 1 + h\colon \quad \varepsilon\Big(p - \frac{\delta\,\mathrm{We}}{1+\delta+\delta h} + \mathrm{We}\,\varepsilon^2\frac{\partial^2 h}{\partial z^2} - 2\varepsilon\frac{\mathrm{Fr}}{\mathrm{Re}}\frac{\partial u}{\partial z}\Big)\frac{\partial h}{\partial z} + \frac{\mathrm{Fr}}{\mathrm{Re}}\Big(\frac{\partial u}{\partial r} + \varepsilon\frac{\partial v}{\partial z}\Big) = \varepsilon p_0\frac{\partial h}{\partial z},$$

$$p - \frac{\delta\,\mathrm{We}}{1+\delta+\delta h} + \mathrm{We}\,\varepsilon^2\frac{\partial^2 h}{\partial z^2} - 2\frac{\mathrm{Fr}}{\mathrm{Re}}\frac{\partial v}{\partial r} + \varepsilon\frac{\mathrm{Fr}}{\mathrm{Re}}\Big(\frac{\partial u}{\partial r} + \varepsilon\frac{\partial v}{\partial z}\Big)\frac{\partial h}{\partial z} = p_0, \tag{7}$$

$$\varepsilon\Big(\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial z}\Big) = v.$$

Representing the disturbed flow in the form $u = V + \varepsilon u'$, $v = \varepsilon^2 v'$, $p = P + \varepsilon p'$, and $h = \varepsilon h'$ and neglecting terms of the order of $\varepsilon^2$ and higher in (5), we obtain the following system of equations for the disturbed quantities (the primes are omitted):

$$\varepsilon\frac{\partial u}{\partial t} + \varepsilon V\frac{\partial u}{\partial z} + \varepsilon v\frac{dV}{dr} = -\frac{\varepsilon}{\mathrm{Fr}}\frac{\partial p}{\partial z} + \frac{\delta}{\mathrm{Re}(1+\delta r)}\frac{\partial u}{\partial r} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial r^2},$$

$$\frac{\partial p}{\partial r} = 0, \qquad \frac{\partial u}{\partial z} + \frac{\delta}{1+\delta r}v + \frac{\partial v}{\partial r} = 0. \tag{8}$$

The boundary conditions (6) remain unchanged: $u = v = 0$ at $r = 0$.

We transfer the boundary conditions (7) from the free surface to the undisturbed level $r = 1$:

$$\frac{\partial u}{\partial r} + \frac{d^2 V}{dr^2}h = 0,$$

$$p + \varepsilon\frac{\partial p}{\partial r}h + \frac{\delta^2\,\mathrm{We}}{(1+\delta)^2}[1 - \mathrm{B}\cos^2(\omega t + \phi)]h + \mathrm{We}\,\varepsilon^2\frac{\partial^2 h}{\partial z^2} - 2\varepsilon\frac{\mathrm{Fr}}{\mathrm{Re}}\frac{\partial v}{\partial r} = 0; \tag{9}$$

$$\frac{\partial h}{\partial t} + (\varepsilon u + V)\frac{\partial h}{\partial z} = v + \varepsilon\frac{\partial v}{\partial r}h. \tag{10}$$

The surface tension is assumed to be strong enough so that the values of $\mathrm{We}\,\varepsilon^2$ and $\mathrm{We}\,\delta^2$ are of the order of unity.

We seek for the solution of system (8) with the boundary conditions (9) and (10) in the form

$$(u, v, p) = \sum_{n=0}^{\infty}\varepsilon^n(u_n, v_n, p_n), \qquad h(z,t) = h(z,t,\varepsilon t) = h(z,t,t_1).$$

Equating the coefficients at the same powers of $\varepsilon$, in the zero approximation we have

$$u_0 = \frac{\mathrm{Re}}{\mathrm{Fr}}rh, \quad v_0 = -\frac{\mathrm{Re}}{\mathrm{Fr}}\frac{r^2}{2}\frac{\partial h}{\partial z}, \quad p_0 = -\mathrm{We}\,\delta^2[1 - \mathrm{B}\cos^2(\omega t + \phi)]h - \mathrm{We}\,\varepsilon^2\frac{\partial^2 h}{\partial z^2}.$$

Substituting the zero-order solution of system (8) into the kinematic condition (10), in the first approximation we obtain

$$\frac{\partial h}{\partial t} + \Big(1 + \frac{\mathrm{Re}}{2\,\mathrm{Fr}}\Big)\frac{\partial h}{\partial z} = 0. \tag{11}$$

With the expression for the flow velocity at the surface of the undisturbed film, it can easily be shown that $\mathrm{Re}/\mathrm{Fr} = 2$. Then, it follows from (11) that, in the first approximation, all disturbances propagate at a velocity twice the velocity at the undisturbed free boundary.

For the next order in $\varepsilon$, from system (8) we obtain

$$\frac{\partial u_0}{\partial t} + V\frac{\partial u_0}{\partial z} + v_0\frac{dV}{dr} = -\frac{1}{\mathrm{Fr}}\frac{\partial p_0}{\partial z} + \frac{S}{\mathrm{Re}(1+\delta r)}\frac{\partial u_0}{\partial r} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u_1}{\partial r^2},$$

$$\frac{\partial p_1}{\partial r} = 0, \qquad \frac{\partial u_1}{\partial z} + \frac{S}{1+\delta r}v_0 + \frac{\partial v_1}{\partial r} = 0, \tag{12}$$

where $S = \delta/\varepsilon = \lambda/R \sim 1$. In this approximation, the boundary conditions at $r = 1$ take the form

$$\frac{\partial u_1}{\partial r} = 0, \quad p_1 + \frac{\partial p_0}{\partial r}\, h - 2\,\frac{\mathrm{Fr}}{\mathrm{Re}}\,\frac{\partial v_0}{\partial r} - 2S\delta^2\,\mathrm{We}[1 - \mathrm{B}\cos^2(\omega t + \phi)]h = 0; \tag{13}$$

$$\frac{\partial h}{\partial t_1} + u_0\,\frac{\partial h}{\partial z} = \frac{\partial v_0}{\partial r}\, h + v_1. \tag{14}$$

The solution of system (12) with the boundary condition (13) are easy to find; this solution, however, is too cumbersome to be presented here, and we give only the final expression for the velocity $v_1$ at the boundary:

$$v_1 = \frac{5}{24}\,\frac{\mathrm{Re}^2}{\mathrm{Fr}}\,\frac{\partial^2 h}{\partial t \partial z} - \frac{1}{3}\,\frac{\mathrm{Re}}{\mathrm{Fr}}\,\mathrm{We}\Big(\delta^2[1 - \mathrm{B}\cos^2(\omega t + \phi)]\frac{\partial^2 h}{\partial z^2} + \varepsilon^2\,\frac{\partial^4 h}{\partial z^4}\Big) - \frac{S}{3}\,\frac{\mathrm{Re}}{\mathrm{Fr}}\,\frac{\partial h}{\partial z} + \frac{3}{40}\,\frac{R^3}{\mathrm{Fr}^2}\,\frac{\partial^2 h}{\partial z^2}.$$

Passing into a reference system moving with a velocity $2 + 2\delta/3$ and substituting the expression for $v_1(1)$ into the kinematic condition (14), we obtain the equation

$$\frac{\partial h}{\partial t_1} + 4h\,\frac{\partial h}{\partial x} + \Big[\frac{8}{15}\,\mathrm{Re} + \frac{2}{3}\,\mathrm{We}\,\delta^2\Big(1 - \frac{\mathrm{B}}{2}\Big) - \frac{\mathrm{We}\,\delta^2}{3}\,\mathrm{B}\cos(2\omega_1 t_1 + \phi)\Big]\frac{\partial^2 h}{\partial x^2} + \frac{2}{3}\,\mathrm{We}\,\varepsilon^2\,\frac{\partial^4 h}{\partial x^4} = 0. \tag{15}$$

Here $h = h(x, t_1)$, $x = z - 2t - 2\delta t/3$, and $\omega_1 = \omega/\varepsilon$.

If the conductor carries a direct current, then, the bracketed time-periodic function in (15) vanishes, and Eq. (15) transforms into the well-known Kuramoto–Sivashinsky equation. In that case, as it follows from (15), if the magnetic Bond number is greater than the critical value $\mathrm{B}^* = 2 + 8\,\mathrm{Re}/(5\,\mathrm{We}\,\delta^2)$, then the constant coefficient at the second derivative is negative. This means that, in this model, a sufficiently intense magnetic field perfectly stabilizes the flow, i.e., there are no unstable disturbances, and all infinitesimal perturbations readily decay with time.

In the present study, we deal with Bond numbers smaller than the critical one. In this case, Eq. (15) can be transformed into an equation for which, at $I_{\mathrm{eff}} = I_0/\sqrt{2}$ ($I_{\mathrm{eff}}$ is the direct current for which the ponderomotive force due to the magnetic field equals the time-mean force at the given alternating current), the neutral wavenumber $\alpha_n$ for infinitesimal perturbations equals unity. It follows from Eq. (15) that in this case the condition

$$4\,\mathrm{Re}/(5\,\mathrm{We}\,\varepsilon^2) = 1 - S^2(1 - \mathrm{B}/2) \tag{16}$$

must be fulfilled. Equation (16) gives a more adequate estimate of the characteristic streamwise length of the disturbances. It follows from this equation that

$$\varepsilon = h_0/\lambda = \delta(1 - \mathrm{B}/2 + 4\,\mathrm{Re}/(5\,\mathrm{We}\,\delta^2))^{1/2}.$$

Making the substitutions $h(x, t_1) = 2\,\mathrm{We}\,\varepsilon^2 H(x, \tau)/3$, $\tau = 2\,\mathrm{We}\,\varepsilon^2 t_1/3$, $\omega_1 = 2\,\mathrm{We}\,\varepsilon^2\Omega/3$, and $C = S^2\,\mathrm{B}/2$, we finally obtain

$$\frac{\partial H}{\partial \tau} + 4H\,\frac{\partial H}{\partial x} + [1 - C\cos(2\Omega\tau + \phi)]\,\frac{\partial^2 H}{\partial x^2} + \frac{\partial^4 H}{\partial x^4} = 0. \tag{17}$$

**2. Solution Algorithm.** Equation (17) with the nonlinear term omitted describes the stability of the initial flow against infinitesimal disturbances. It should be remembered that the neutral wavenumber $\alpha_n$ equals unity for $C = 0$. Consequently, it is the disturbances with wavenumbers $\alpha < 1$ that are unstable here.

To analyze the nonlinear periodical solutions, we numerically solved Eq. (17). The solution was represented as a Fourier series in space with time-dependent harmonics:

$$H(x, \tau) = \sum_{n=-\infty}^{\infty} H_n(\tau)\exp(in\alpha x). \tag{18}$$

Since the values of $H$ are real, the relation $H_{-n} = H_n^*$ holds for the harmonics $H_n$ (the superscript asterisk denotes a complex-conjugate quantity).

Substituting (18) into (17), we obtain an infinite system of nonlinear ordinary differential equations for the harmonics $H_n$. Truncating series (18), i.e., assuming that all harmonics beginning from a certain number $N$ equal zero, we arrive at a finite approximation of the system:

$$\frac{dH_n}{d\tau} = \alpha^2 n^2\{[1 - C\cos(2\Omega\tau + \phi)] - \alpha^2 n^2\}H_n - 4i\alpha n \sum_{m=N-n}^{N} H_m H_{n-m}, \quad n = 0, 1, \ldots, N. \tag{19}$$

It follows from (19) that the equation for the zero (fundamental) harmonics is trivial, i.e., it holds for an arbitrary constant value of $H_0$. In what follows, we are going to construct solutions in which this constant equals zero.

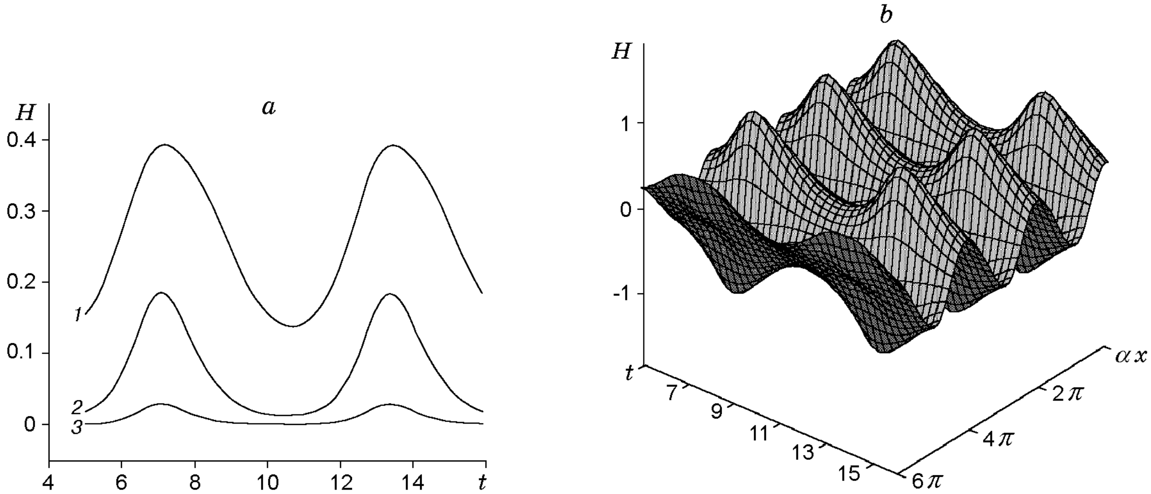


Fig. 1

Separating the real and imaginary parts in (19), we have a system of $2N$ ordinary differential equations for $2N$ unknown functions.

System (19) was solved numerically using a fifth-order Runge–Kutta method with an automatic choice of the integration step and controlled accuracy. The total number of harmonics was taken such that to satisfy the relation $|H_N|/\max|H_n| < 10^{-4}$. The main results were obtained for $N = 10$. To verify the solutions obtained, in some cases we also solved the system with $N$=25. A direct comparison revealed only a small difference between the results, no greater than in the fourth or fifth decimal digit.

**3. Results.** As is noted above, Eq. (17) with a direct current ($C = 0$) transforms into the well-known Kuramoto–Sivashinsky equation, which was the subject matter of many previous studies. It follows from these studies that the solution pattern of the Kuramoto–Sivashinsky equation is extremely rich. For instance, Nepomnyashchii [4] showed that, for $\alpha = 1$, a family of periodic solutions of the Kuramoto–Sivashinsky equation appears, branching off from its trivial solution, which can be extended to the wavenumber $\alpha = 0.4979$ (the so-called first family of solutions). In our previous studies [5, 6] we showed how a succession of bifurcations, which gives rise to increasingly more complex solutions, emerges. In view of the great complexity of possible solutions of the Kuramoto–Sivashinsky equation, here we restrict our consideration to such solutions of Eq. (17) for which the initial disturbances for $C = 0$ develop into steady-state traveling solutions of the Kuramoto–Sivashinsky equation.

In the computations described below, the real and imaginary parts of the first harmonic $H_1$ were assumed to equal 0.5 and 0, respectively. The initial amplitudes of all other harmonics were put equal to zero.

As the computations showed, even in that domain of parameters where the structure of solutions of the Kuramoto–Sivashinsky equation is rather simple, the wave pattern for Eq. (17) becomes more diversified. The reason is that the amplitudes of harmonics oscillate in time under the action of the periodic magnetic field. These oscillations are largely affected by both the amplitude and frequency of the alternating current. The effect of the phase $\phi$ is much less pronounced; this effect is observed only in the initial transition regime, for which reason we assumed that $\phi = 0$ in all computations described below.

Figure 1a shows the time evolution of the amplitudes of the first three harmonics (curves 1–3) for the wavenumber $\alpha = 0.8$, $C = 1$, and $\Omega = 0.5$. The amplitudes of all other harmonics were within several percents of the highest amplitude of the third harmonic. Since the first harmonic is always more intense than the second one, the shape of the wave profile is almost sinusoidal. In turn, as follows from Fig. 1a, at least the first two harmonics oscillate at $t > \pi/\Omega$ according to a quasi-sinusoidal law. Figure 1b shows the time sweep of the wave profile for the same values of determining parameters. In this case, the shape of the wave profile along the $x$ and $t$ coordinates is almost sinusoidal.

As the computations showed, for wavenumbers from a close vicinity of the neutral wavenumber, the solutions of this type are typical, provided that $C/\Omega < 1$, which condition is quite realistic because, on the one hand, the lower the amplitude, the weaker the effect due to the alternating magnetic field, and on the other hand, the influence of the alternating component of the magnetic force at a sufficiently high frequency is insignificant.

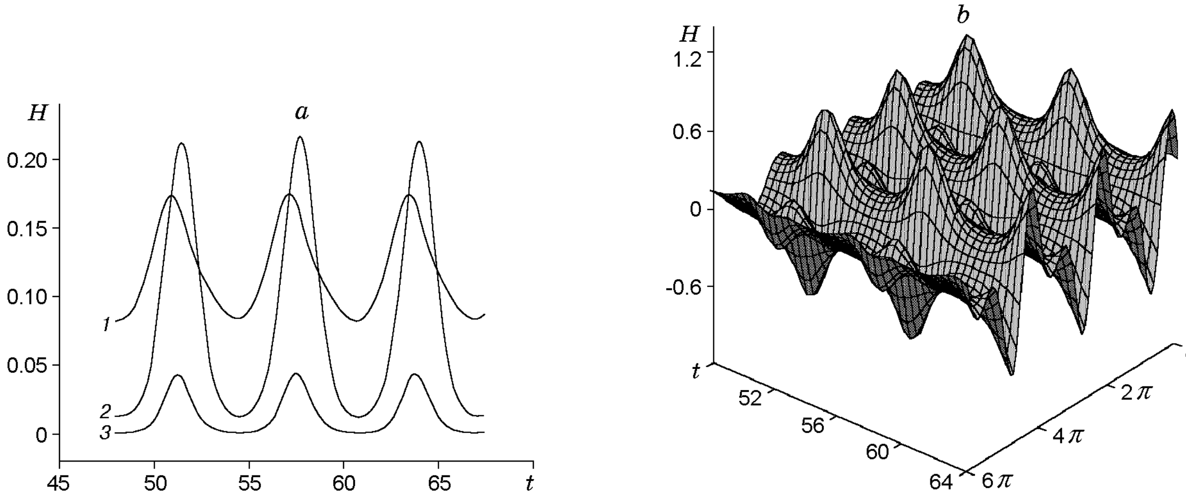


Fig. 2

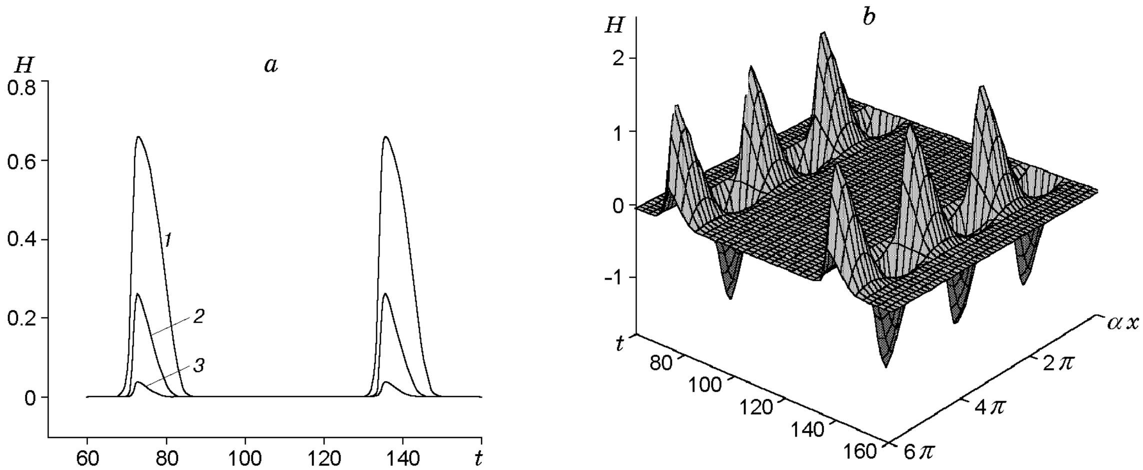


Fig. 3

For the indicated relation between the amplitude $C$ and frequency $\Omega$, as we move further along the wavenumber $\alpha$ into the linear instability region for the Kuramoto–Sivashinsky equation, the wave profile becomes essentially nonsinusoidal. The results plotted in Fig. 2 were obtained for $\alpha = 0.6$, $C = 1$, and $\Omega = 0.5$. As is shown in Fig. 2a, the second harmonic (curve 2) for the above values of governing parameters becomes comparable with the first one (curve 1) and sometimes even exceeds it. Such a behavior of the second harmonic is due to the fact that, during these time intervals, the second harmonic also enters the instability region together with the first harmonic. Indeed, it may be argued that, unlike the case of the Kuramoto–Sivashinsky equation, the linear instability region for the solutions of Eq. (17) in a sense varies in time. In particular, if the frequency $\Omega$ is low, then we may consider the phase at the coefficient at the second derivative in Eq. (17) as being almost constant during a fairly long period of time. Then, the local linear instability region lies in the wavenumber interval

$$0 < \alpha < [1 - C\cos(\Omega\tau + \phi)]^{1/2}. \tag{20}$$

Thus, in this case, at the moments when the second harmonic ($\alpha = 1.2$) falls in the interval (20) together with the first harmonic ($\alpha = 0.6$); its amplitude rapidly grows both due to the self-amplification caused by linear instability and due to the nonlinear action of the first harmonic. In this situation, the third harmonic always remains in the linear stability region, being, therefore, weaker than the first two harmonics (curve 3 in Fig. 2a).

The time sweep of the wave profile is shown in Fig. 2b. In this case, the shape of the wave profile along the $x$ and $t$ coordinates is seen to differ considerably from a sinusoidal one.

If the frequency $\Omega$ is low (of the order of 0.1 or smaller), then even for wavenumbers close to the neutral wavenumber ($\alpha = 1$), the fluctuations of the solution in time grow in value with increasing amplitude of the alternating current, becoming increasingly more nonsinusoidal (Fig. 3). The results shown in Fig. 3 were obtained for $C = 2$, $\Omega = 0.05$, and $\alpha = 0.95$. In this case, these appear regions where the surface of the fluid remains undisturbed during a fairly long time interval. The explanation to this behavior of the solution is analogous to that previously given in [7] for the case of a film flow down a vibrating vertical plate. The latter is related to the fact that, for low frequencies $\Omega$ and sufficiently large amplitudes $C$, the first harmonic at some moments leaves the local linear instability region (20). As the wavenumber $\alpha$ enters the linear stability region, the disturbance amplitude rapidly vanishes. As a result, there are no disturbances on the film during a fairly long period till the wavenumber $\alpha$ enters the local instability region (20) once again. At this moment, the signal rapidly increases, and the process recurs over and over.

Figure 3a illustrates the time evolution of the first three harmonics (curves 1–3). The time intervals in which the harmonics exhibit nonzero amplitudes are rather narrow. Since the first harmonic dominates at the adopted values of governing parameters, the spatial waveform of the wave profile at these moments is also close enough to a sinusoidal one (Fig. 3b).

**4. Conclusions.** Equation (17) derived in the present work may be used to study the effect of a time-dependent magnetic field on weakly nonlinear disturbances in a film flow of a ferromagnetic viscous fluid. The character of the waves is largely influenced both by the frequency of the field and by its amplitude. The computations show that, as a consequence, nontrivial wave regimes may emerge, which do not occur in the case of a freely falling fluid film.

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